**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: µ = 45

σ = 8

Complete time = 1 hr

Leftover time after drop-off(x) = 50mins

P(50) will be 0.7340145

To find Z-score,

Z = (X - µ) / σ

Z = (50 – 45)/ 8

So using R, as confidence interval is not given we use pnorm function or else we use qnorm function.

> pnorm(50,45,8)

[1] 0.7340145

P(50) will be 0.7340145

Out of 1 hour, the probability of leftover 10 mins will be

P(10) = 1 – P(50)

= 1 – 0.7340145

= 0.265

Therefore the probability that the service manager cannot meet his commitment will be 0.2676(B)

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans: False.

According to Normal distribution, it follows a symmetry and a bell-shaped curve with Mean= Median= Mode. So according to my understanding the employees will be more near the Mean that is between 38 to 44 and not older than 44.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: True

1. If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid*normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: Consider X1 = {1, 2, 3}, Mean = 2, Variance = 1

X2 = {3, 2, 1}, Mean = 2, Variance = 1

2 X1 = 2{1, 2, 3} = {2, 4, 6}, Mean = 4, Variance = 4

X1 + X2 = {1, 2, 3} + {3, 2, 1} = {4, 4, 4}, Mean =4, Variance= 0

Both the variables that is X1 and X2 are given normal random variables but when we compare 2X1 and X1+X2, the Mean values are same but X1+X2 has zero variance.

1. Let X ~ N(100, 202). Find two values, *a* and*b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: Given µ = 100

σ²=20

P(random variable i.e., X) = 0.99

From Z-table for 99% significance level, i.e., at 0.995, X = 2.58

Interval Estimate = Point Estimate ± Margin of Error

= Mean ± Z(1-α) σ/√n

= 100 ± 2.58(20)

= [48.4, 151.6] = D

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Profit1, Mean = 5, Variance = 3

Profit2, Mean = 7, Variance = 4

1$ = 45Rs

Profits = 1million

A. Confidence Interval = 95%

Inorder to find Z-score at 95% confidence interval, Using Z-score table Z= 1.96

Converting both the profits from millions to rupees,

* (45\*12, 45\*25) = (540, 225²)

Therefore to find the specific Rupee range at 95% confidence interval,

Interval Estimate = Point Estimate ± Margin of Error

= 540 ± 1.96(225) = [99, 981]

B. 5th Percentile will be profit at 90% Confidence Interval

Z-score at 90% confidence interval will be 1.64

= 540 – 1.64(225) = 171

We are finding only one value not the range because we are finding the leftover % from 100%.

C. To find out the larger probability of making loss out of two divisions,

For Profit1, (0-5)/3 = -1.6666667

For Profit2, (0-7)/4 = -1.75

2nd Division i.e., Profit 2 has larger probability of making loss.